chpt. 7
Q1 - 15
Q4
Part . 16
7.17 / 8.c

9
Q3 / 6
18.

BIRZEIT UNIVERSITY

Mathematics Department

STAT236 – Second-Hour Exam - Fall 2011

Student Name:

Section:

1	Mohammad Madiah	MW 14:00 – 15:20
2	Tareq Sadeq	SMW 10:00 – 10:50
3	Tareq Sadeq	SMW 13:00 – 13:50
4	Hani Kabajah	SMW 12:00 – 12:50
5	Hani Kabajah	SMW 09:00 – 09:50
6	Maher Abdellatif	TR 09:30 – 10:50
7	Hani Kabajah	SMW 08:00 – 08:50

13
09
08.5
30.5 / 40

Formulas:

- Binomial: $f(x)=\binom{n}{x}p^x(1-p)^{n-x}$, $E(x)=np$, $\sigma=\sqrt{np(1-p)}$
- Poisson: $f(x)=\dfrac{\mu^x e^{-\mu}}{x!}$
- Exponential: $F(x_0)=P(x\le x_0)=1-e^{-\frac{x_0}{\mu}}$
- Standard error of the sample mean: $\sigma_{\bar{x}}=\dfrac{\sigma}{\sqrt{n}}\sqrt{\dfrac{N-n}{N-1}}$ (Finite population)

  $\sigma_{\bar{x}}=\dfrac{\sigma}{\sqrt{n}}$ (Infinite population)
- Standard error of the sample proportion: $\sigma_{\bar{p}}=\sqrt{\dfrac{p(1-p)}{n}}\sqrt{\dfrac{N-n}{N-1}}$ (Finite population)

  $\sigma_{\bar{p}}=\sqrt{\dfrac{p(1-p)}{n}}$ (Infinite population)

x	f(x)	x·f(x)	x²	x²·f(x)
1	1/10	0.1	1	0.1
4	4/10	1.6	16	6.4
5	5/10	2.5	25	12.5
Total	10/10			

$19 - 17.64$

Question 1: Find the correct answers

$\sigma = E(x^2) - E(x)^2$

If X is a discrete random variable with a probability function. Questions 1-2.

$$f(x) = \frac{x}{10}, \quad x = 1, 4, 5$$

$\Sigma f(x)\, x = \mu$

1. Find E(X).
   a. 4.2 b. 6 c. 3.8 d. 1.36

$\Sigma (x-\mu)^2 f(x)$

2. Find Var(X).
   a. 4.2 b. 1.56 c. 0.89 d. 1.36

$P(X \geq 2) = 1 - f(1) + f(0)$

A sample of 10 Palestinians is randomly and independently selected. If 30% of the Palestinian population are unemployed, Questions 3 - 5 np

$n = 10$
$p = 0.3$
$n \cdot p =$

3. What is the mean number of unemployed persons?
   a. 2 b. 8 c. 1.6 d. 3

$f(5) = \binom{10}{5}(0.3^5)(1-0.3)^{10-5}$

4. What is the probability that 5 persons are unemployed?
   a. 0.103 b. 0.0264 c. 0.107 d. 0.5

$252 \cdot 61236$

5. What is the probability that at least 2 persons are unemployed?
   a. 0.121 b. 0.879 c. 0.3754 d. 0.6246

$1 - (f(0) + f(1))$

6. An airport administration desires to study the waiting time in a line at check-in desk. The time follows an exponential distribution with a mean of 10 minutes. The probability that the waiting time is less than 15 minutes is:
   a. 0.2231 b. 0.7135 c. 0.2865 d. 0.7769

$1 - e^{\frac{x}{\mu}}$ $\mu = 10$

$(X < 15)$
$1 - e^{-X/\mu}$
$1 - e^{-15/10}$

A manufacturer produces keys at the rate of 10 keys per hour. Questions 7 - 9

$(\mu = 10 \to$ per hour$)$

7. What is the probability that the manufacturer produces exactly 8 keys in an hour?
   a. 0.063 b. 0.0076 c. 0.0413 d. 0.11

$\frac{\mu^x e^{-\mu}}{x!}$

8. What is the probability that the manufacturer produces exactly 2 keys in 15 minutes?
   a. 0.2565 b. 0.3 c. 0.0076 d. 0.2137

$\mu = 2.5$ in 15 min $f(Y) = 10^8 e^{-10}$

9. What is the probability that the time needed to produce a key is $\boxed{\text{more}}$ than 10 minutes?
   a. 0.1889 b. 0.2 c. 0.1 d. 0.3679

$\frac{60}{10} = 6 = \mu$ $e^{-\frac{x}{\mu}} = e^{-\frac{10}{6}} = 0.1889$

10. A computer randomly selects numbers between 4 and 10 with uniform probability distribution. What is the probability that a number will have a value of at least 5?
   a. 0.25 b. 0.33 c. 0.833 d. 0.625

$\frac{a}{b-a}$ $5/6$ $\left(\frac{5}{6}\right)$ 13 $\left(\frac{b-x}{b-a}\right)$ $\frac{5}{6}$ $10 \to 60$ min, 15 min

$1 - (0.121 + 0.03)$

$P(X \geq 2) \Rightarrow 1 - f(1) + f(0)$

$\frac{2.5^2 e^{-2.5}}{2!}$ $1 - \binom{10}{1}(0.3^1)(1-0.3)^{10-1}$

$N(900, 40)$

The wages of employees follow a normal distribution with a mean of $900 and a standard deviation of $40. Questions 11 – 14

11. What is the probability that a selected employee will have a wage greater than $870?
a. 0.25 b. 0.8944 c. 0.1056 d. 0.7734 $Z = -0.75$ P(.75)

12. What is the probability that a selected employee will have a wage between $1000 and $1018?
a. 0.9876 b. 0.9938 c. 0.0062 d. 0.0046 $2.5 < z < 2.95$

13. What is the probability that a selected employee will have a wage in the top 5% of wages?
a. 0.9 b. 0.05 c. 0.1 d. 0.2 focus $0.09 = \frac{X - 900}{40}$

14. An employee has a wage among the top 5% of wages if her wage is higher than:
a. $965.79 b. $900 c. $940.5 d. $951.26

965.8

15. Palestine Central Bureau of Statistics selects a sample, where 20% of the sample individuals are unemployed. This is the same as the population unemployment rate. This sampling method is called:
a. Simple random sampling
b. Systematic sampling
c. Stratified sampling
d. Convenience sampling
e. Judgment sampling

binomial

chapter not in

16. A professor at BZU studies the determinants of students grades. The professor asks some students to select random samples of BZU students and let them fill a questionnaire. This sampling method is called:
a. Simple random sampling
b. Systematic sampling
c. Stratified sampling
d. Convenience sampling
e. Judgment sampling

Question 2:

finite

State whether each of the following is a discrete or continuous random variable?

D a. The number of customers arriving at a bank: ... ~~continuous~~ discrete → infinite
C b. The wages of employees at a company: ... discrete → finite ...
D c. The number of customers contacted per day by a company: ... → discrete → finite
C d. The time passed during an interviews: ... continuous ...

Question 3:

Assume you know that 10% of STAT236 students fail the course. A class of 100 students is selected.

$p = 0.1$ $n = 100$ $S = .9$ $F = .1$

a) What is the mean number of failures in the sample?

$B(100, 0.1)$ n p $\to \mu = n \cdot p$ ~~$(100 * 0.1)$~~ $= 10$

$\mu = pn = .1 * 100 = 10$

$N(\mu, \sigma) \to 3$ 10

b) What is the standard deviation of failures?

$\sigma = \sqrt{n \cdot p(1-p)}$ $\sigma = 3$ $\sqrt{pn(1-p)}$

$= \sqrt{10(1-0.1)}$ $\sqrt{9} = 3$

c) Check if the conditions of normal approximation of binomial probabilities are satisfied.

it is normal approximation because ① $n \cdot p \geq 5$ $10 \geq 5$

$np \geq 5$ $n(1-p) > 5$

② $n(1-p) \geq 5$ $100(1-0.1) \to 90 \geq 5$

d) What is the approximate probability that at most 6 students will fail in the sample of 100 students?

$N(10, 3)$

$n = 100$

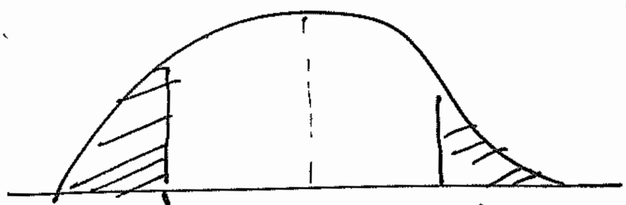
$P(X \leq 6) = P\left(Z < \frac{6.5 - 10}{3}\right) = \boxed{1.17}$

$= P(Z < -1.5) \approx P(Z > 1.5)$

$= 1 - P(Z < 1.5)$

$= 1 - (0.9332)$

$= \boxed{0.0668}$



$z = -1.17$ 1.17

$1 - P(z = 1.17)$

$1 - (.879)$

$= .121$

$X \geq 10 \to 9.5$ $X > 10 \to 10.5$ $X < 10 \to$

8.5/10

Question 4:

In 2006, the participation rate in Palestinian elections was 70%. A survey of 100 voters was conducted to estimate the proportion of those who participated at elections.

$p = 0.7$
$n = 100$

a) What is the probability that one randomly selected individual will have participated in elections?

$p(x=1) \Rightarrow \frac{1-0.7}{0.046}$ $= 6.52$

$\bar{p} - p$

$\sqrt{\frac{p(1-p)}{n}}$

$\sqrt{\frac{(0.7)(1-0.7)}{100}}$

$\sigma_{\bar{p}} = \boxed{0.046}$

$\boxed{6.52} = 0$

لأنه أكبر حد 3
أصغر حد -3

b) Find the expected value of the sample proportion.

$E(\bar{p}) = \mu_{\bar{p}} = p$
$= 0.7$

c) Find the standard error of the sample proportion.

$\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}}$
$= \sqrt{\frac{0.7(1-0.7)}{100}} = \boxed{0.046}$

d) What is the probability that the sample proportion will be within ±0.08 of the population proportion?

$P(-0.08 < |\bar{p} - p| < +0.08)$

$P(p - 0.08 < \bar{p} < p + 0.08)$

$P(0.7 - 0.08 < \bar{p} < 0.7 + 0.08)$

$P(0.62 < \bar{p} < 0.78)$

$P\left(\frac{0.62 - 0.7}{0.046} < \bar{p} < \frac{0.78 - 0.7}{0.046}\right)$

$P\left(\frac{-0.08}{0.046} < \bar{p} < \frac{0.08}{0.046}\right)$

$P(-1.74 < \bar{p} < 1.74)$

كونها
z score

$P(z < 1.74) - P(z < -1.74)$

$0.9591 - (1 - P(z < 1.74))$

$0.9591 + 0.9591 - 1$

$= \underline{0.9182}$

Part: Show all your work

16. A life insurance company has determined that each week an average of seven claims is filed in one of its branch.

5 points a. What is the probability that during the next week exactly seven claims will be filed?

5 points b. What is the expected number of claims in two weeks?

a) $P(X=7) = \frac{e^{-7}\, 7^{7}}{7!} = 0.149 \approx 0.15$

b) $E(X)_{2\text{ weeks}} = 2\, E(X)_{1\text{ week}} = 2(7) = 14$

17. The average price of personal computers manufactured by MNM Company is $1,200 with a standard deviation of $220. Furthermore, it is known that the computer prices manufactured by MNM are normally distributed. DO NOT ROUND YOUR NUMBERS.

5 points a. What is the probability that a randomly selected computer will have a price of at least $1,530?

5 b. What are the minimum and the maximum values of the middle 95% of computer prices?

5 c. If 513 of the MNM computers were priced at or below $647.80, how many computers were produced by MNM?

a) $P(X \geq 1530) = P\left(Z \geq \frac{1530-1200}{220}\right) = P(Z \geq 1.5)$

$= 0.5 - 0.4332 = 0.0668$

b) $Z_{0.025} = 1.96 = \frac{X - 1200}{220} \Rightarrow X = 1200 + 1.96(220) =$

$X = 1200 + 431.2 = \boxed{1631.2}$

$Z_{.975} = -1.96 = \frac{X-1200}{220} \Rightarrow X = 1200 - 431.2 = \boxed{768.8}$

$\Rightarrow$ Min. $= 768.8$, Max $= 1631.2$

c) $\frac{513}{N} = P(X \leq 647.8) = P\left(Z \leq \frac{647.8-1200}{220}\right)$

$= P(Z \leq -2.51) = 0.5 - (0.494) = 0.006$

$\Rightarrow N = 513/(0.006) = \boxed{85,500}$

$P(X>12) = P(X \geq 12.5) = P(Z > \frac{12.5-8}{2.53}) = P(Z > \cancel{1.78})$ 1.50

(a) $= 0.5 - \cancel{.4625}$ .4162 $= \boxed{\cancel{.0375}} = 0.0838$

$\mu = n\pi = 40(0.2) = 8$ $\sigma_p = \sqrt{\frac{(0.2)(0.8)}{40}} = 0.0633$

$\sigma = \sqrt{n\pi(1-\pi)} = \sqrt{8(0.8)} = 2.53$

18. Twenty percent of the applications received for a particular position are rejected. What is the probability that among the next 40 applications,

a. More than 12 will be rejected? 0.87%

b. Determine the expected number of rejected applications and its variance.

$\boxed{P(x \geq 11.5)}$??

points a) $P(x>12) = P(p > \frac{12}{40}) = P(p > 0.3) = P(Z > \frac{0.3-0.2}{0.0633})$

$= P(Z > 1.5798) = 0.5 - 0.4429 = \boxed{0.0571}$

points b) $E(X) = n\pi = 40(0.2) = 8$

$V(X) = n\pi(1-\pi) = 8(0.8) = 6.4$

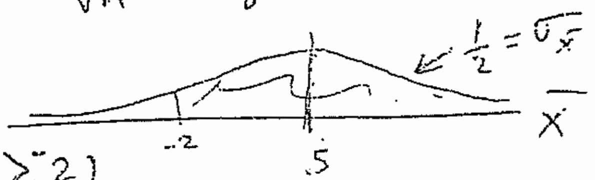
19. Students of a large university spend an average of $5 a day on lunch. The standard deviation of the expenditure is $3. A simple random sample of 36 students is taken.

5 points a. What are the expected value, standard deviation, and shape of the sampling distribution of the sample mean?

5 " b. What is the probability that the sample mean will be at least $4?

(5) a) $E(\bar{X}) = \mu = 5$, $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{3}{6} = \frac{1}{2}$

shape : Normal



b) $P(\bar{X} \geq 4) = P(Z \geq \frac{4-5}{\frac{1}{2}}) = P(Z \geq 2)$

(5) $= \cancel{0.5 - 0.4772} = \boxed{\cancel{0.0228}}$

$0.5 + P(0 < Z < 2) = 0.5 + 0.4772 = 0.9772.$

20. Ten percent of the items produced by a machine are defective. A random sample of 100 items is selected and checked for defects.

5 points a. Determine the standard error of the sample proportion.

5 points b. What is the probability that the sample will contain more than 2.5% defective units?

(5) a) $\sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}} = \sqrt{\frac{(0.1)(0.9)}{100}} = 0.03$

b) $P(p > 0.025) = P(Z > \frac{0.025 - 0.1}{0.03})$

(5) $= P(Z > -2.5) = 0.5 + 0.4938 = \boxed{0.9938}$

$P(x > 11.5)$